A major open problem in Bounded Arithmetic is to find suitable characterizations of the Σ^b_i-definable search problems of the theories T^k_2 for $i < k$, which might allow a separation of the hierarchy of relativized theories $T^k_2(\alpha)$ by formulas of lower quantifier complexity.

In this paper, a new characterisation of the Σ^b_{i+1}-definable search problems of the theories T^{k+1}_2, for $i \leq k$ is given. These characterisations are in terms of a generalisation of the class PLS of polynomial local search problems to the polynomial time hierarchy, the Π^b_k-PLS problems with Π^b_i-goals also introduced in the present paper. They are proved using the method of proof notations developed by the first author with Aehlig [1].

Essentially the same results have also been shown by a more traditional witnessing argument in a companion paper [2] by the authors.

References

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[2] A. Beckmann and S. R. Buss, Polynomial local search in the polynomial hierarchy and witnessing in fragments of bounded arithmetic, J. Math. Log. **9** (2009), no. 1, 103–138. MR2665783